Speech Signal Processing

David Weenink

Institute of Phonetic Sciences
University of Amsterdam

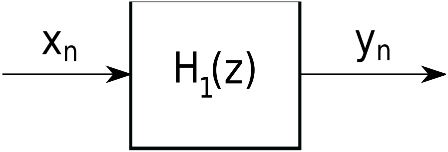
First semester 2007

Digital Filters, Pre-emphasis, Formant Filters

Digital Filters

What is a digital filter?

"An algorithm that calculates with sample values"

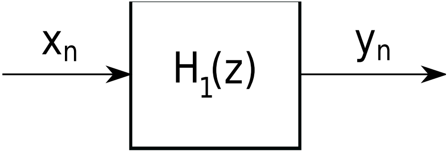


- Filter/machine $H_1(z)$ that:
- Given input value x_n
- Calculates output value y_n (no delay)
- One value per sampling period T (the clock)

Digital Filters

What is a digital filter?

"An algorithm that calculates with sample values"

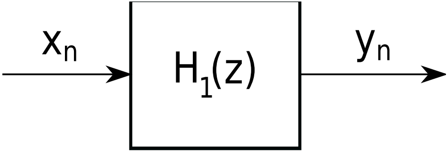


- Filter/machine $H_1(z)$ that:
- Given input value x_n
- Calculates output value y_n (no delay)
- One value per sampling period T (the clock)

Digital Filters

What is a digital filter?

"An algorithm that calculates with sample values"

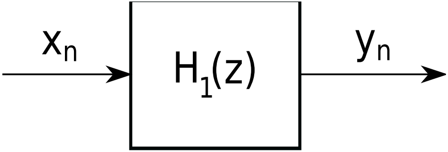


- Filter/machine $H_1(z)$ that:
- Given input value x_n
- Calculates output value y_n (no delay)
- One value per sampling period T (the clock)

Digital Filters

What is a digital filter?

"An algorithm that calculates with sample values"



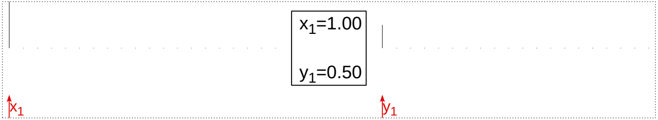
- Filter/machine $H_1(z)$ that:
- Given input value x_n
- Calculates output value y_n (no delay)
- One value per sampling period T (the clock)

Usage?

- Pre-emphasis od speech signals
- Formant synthesis
- Filtering in the time domain
- ...

Simple Integrator

Filter: $y_n = 0.5x_n + 0.5x_{n-1} (n = 1, 2, \dots)$

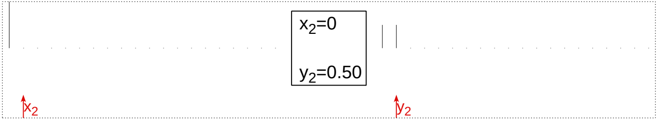


Calculations

- $y_1 = 0.5x_1 + 0.5x_0 = 0.5 \times 1 + 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 0 + 0.5 \times 1 = 0.5$
- $y_3 = 0.5x_3 + 0.5x_2 = 0.5 \times 0 + 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 + 0.5x_3 = 0.5 \times 0 + 0.5 \times 0 = 0$

Simple Integrator

Filter: $y_n = 0.5x_n + 0.5x_{n-1} (n = 1, 2, \ldots)$

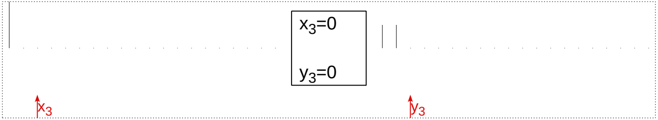


Calculations

- $y_1 = 0.5x_1 + 0.5x_0 = 0.5 \times 1 + 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 0 + 0.5 \times 1 = 0.5$
- $y_3 = 0.5x_3 + 0.5x_2 = 0.5 \times 0 + 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 + 0.5x_3 = 0.5 \times 0 + 0.5 \times 0 = 0$

Simple Integrator

Filter: $y_n = 0.5x_n + 0.5x_{n-1} (n = 1, 2, \ldots)$

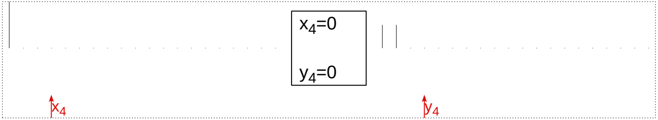


Calculations

- $y_1 = 0.5x_1 + 0.5x_0 = 0.5 \times 1 + 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 0 + 0.5 \times 1 = 0.5$
- $y_3 = 0.5x_3 + 0.5x_2 = 0.5 \times 0 + 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 + 0.5x_3 = 0.5 \times 0 + 0.5 \times 0 = 0$

Simple Integrator

Filter: $y_n = 0.5x_n + 0.5x_{n-1} (n = 1, 2, \ldots)$

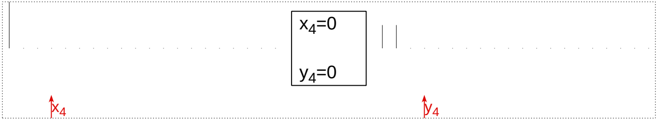


Calculations

- $y_1 = 0.5x_1 + 0.5x_0 = 0.5 \times 1 + 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 0 + 0.5 \times 1 = 0.5$
- $y_3 = 0.5x_3 + 0.5x_2 = 0.5 \times 0 + 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 + 0.5x_3 = 0.5 \times 0 + 0.5 \times 0 = 0$

Simple Integrator

Filter: $y_n = 0.5x_n + 0.5x_{n-1}(n = 1, 2, \dots)$



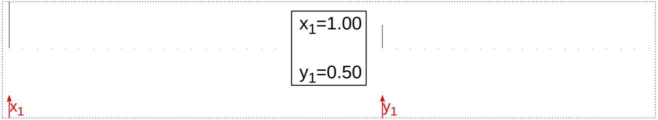
Calculations

- $y_1 = 0.5x_1 + 0.5x_0 = 0.5 \times 1 + 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 + 0.5x_1 = 0.5 \times 0 + 0.5 \times 1 = 0.5$
- $y_3 = 0.5x_3 + 0.5x_2 = 0.5 \times 0 + 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 + 0.5x_3 = 0.5 \times 0 + 0.5 \times 0 = 0$

If $x_n = (1, 0, 0, 0, \dots)$ then $y_n = (0.5, 0.5, 0, 0, \dots)$
(Impulse response)

Simple Differentiator

Filter: $y_n = 0.5x_n - 0.5x_{n-1}(n = 1, 2, \ldots)$

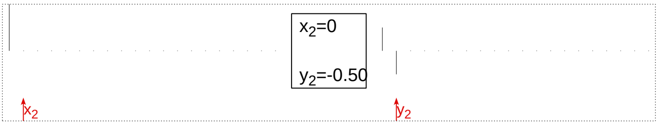


Calculations

- $y_1 = 0.5x_1 - 0.5x_0 = 0.5 \times 1 - 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 - 0.5x_1 = 0.5 \times 0 - 0.5 \times 1 = -0.5$
- $y_3 = 0.5x_3 - 0.5x_2 = 0.5 \times 0 - 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 - 0.5x_3 = 0.5 \times 0 - 0.5 \times 0 = 0$

Simple Differentiator

Filter: $y_n = 0.5x_n - 0.5x_{n-1} (n = 1, 2, \dots)$

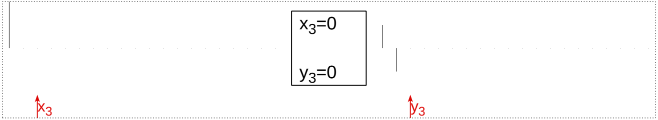


Calculations

- $y_1 = 0.5x_1 - 0.5x_0 = 0.5 \times 1 - 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 - 0.5x_1 = 0.5 \times 0 - 0.5 \times 1 = -0.5$
- $y_3 = 0.5x_3 - 0.5x_2 = 0.5 \times 0 - 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 - 0.5x_3 = 0.5 \times 0 - 0.5 \times 0 = 0$

Simple Differentiator

Filter: $y_n = 0.5x_n - 0.5x_{n-1} (n = 1, 2, \dots)$

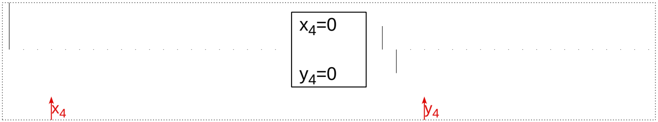


Calculations

- $y_1 = 0.5x_1 - 0.5x_0 = 0.5 \times 1 - 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 - 0.5x_1 = 0.5 \times 0 - 0.5 \times 1 = -0.5$
- $y_3 = 0.5x_3 - 0.5x_2 = 0.5 \times 0 - 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 - 0.5x_3 = 0.5 \times 0 - 0.5 \times 0 = 0$

Simple Differentiator

Filter: $y_n = 0.5x_n - 0.5x_{n-1} (n = 1, 2, \dots)$

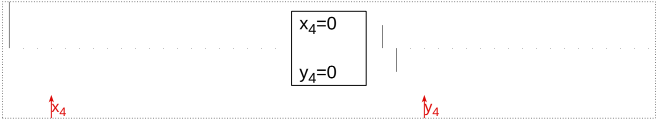


Calculations

- $y_1 = 0.5x_1 - 0.5x_0 = 0.5 \times 1 - 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 - 0.5x_1 = 0.5 \times 0 - 0.5 \times 1 = -0.5$
- $y_3 = 0.5x_3 - 0.5x_2 = 0.5 \times 0 - 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 - 0.5x_3 = 0.5 \times 0 - 0.5 \times 0 = 0$

Simple Differentiator

Filter: $y_n = 0.5x_n - 0.5x_{n-1} (n = 1, 2, \ldots)$



Calculations

- $y_1 = 0.5x_1 - 0.5x_0 = 0.5 \times 1 - 0.5 \times 0 = 0.5$
- $y_2 = 0.5x_2 - 0.5x_1 = 0.5 \times 0 - 0.5 \times 1 = -0.5$
- $y_3 = 0.5x_3 - 0.5x_2 = 0.5 \times 0 - 0.5 \times 0 = 0$
- $y_4 = 0.5x_4 - 0.5x_3 = 0.5 \times 0 - 0.5 \times 0 = 0$

If $x_n = (1, 0, 0, 0, \ldots)$ then $y_n = (0.5, -0.5, 0, 0, \ldots)$
(Impulse response)

Pre-Emphasis Filter 1

Filter: $y_n = x_n - a x_{n-1}$, where $a \in [0, 1]$
What is the spectrum?
take Z-transform on the left and the right sample sequences:

$$\begin{aligned}
\sum_k y_k z^{-k} &= \sum_k \{x_k && -a x_{k-1}\} z^{-k} \\
Y(z) &= \sum \{x_k z^{-k} && -a \sum_k x_{k-1} z^{-k} \\
&= \sum \{x_k z^{-k} && -a \sum_k x_{k-1} z^{-k+1-1} \\
&= \sum \{x_k z^{-k} && -a z^{-1} \sum_k x_{k-1} z^{-(k-1)} \\
&= X(z) && -a z^{-1} X(z)
\end{aligned}$$

The

simplification: $Y(z) = (1 - a z^{-1}) X(z)$
We define $H(z) = Y(z)/X(z)$ as the filter's "spectrum"

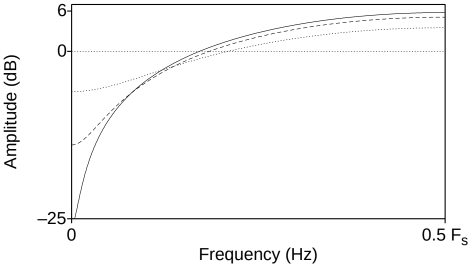
Pre-Emphasis Filter 2

Filter: $y_n = x_n - a x_{n-1}$, where $a \in [0, 1]$
Spectrum: $H(z) = 1 - az^{-1}$,
this equals the sum of the spectrum of $\delta(n)$, $H(z) = 1$,
and the spectrum of $-a\delta(n-1)$, $H(z) = -az^{-1}$!
1em]
$|H(z)|$ shows the filter's amplitude response. We now write $z = e^{+2\pi i f T}$ (f values are "equidistant")

$$\begin{aligned} |H(f)| &= \sqrt{H(f)H^*(f)} = \sqrt{(1 - ae^{-2\pi i f T})(1 - ae^{+2\pi i f T})} \\ &= \sqrt{1 + a^2 - a(e^{+2\pi i f T} + e^{-2\pi i f T})} \\ &= \sqrt{1 + a^2 - 2a\cos 2\pi f T} \end{aligned}$$

Pre-Emphasis Filter Response

$$
\begin{aligned}
20\log|H(f)| &= 20\log\sqrt{1+a^2-2a\cos 2\pi fT} \\
&= 10\log(1+a^2-2a\cos 2\pi fT)
\end{aligned}
$$



- $a = 0.50$ (dotted)
- $a = 0.80$ (dashed)
- $a = 0.95$ (plain)

Extremes: $20\log H(F_s/2)/H(0) = 20\log\frac{1+a}{1-a}$

Digital Filtering in the Time-Domain

Most general linear digital filter formula:

$$y_n = \sum_{k=0}^{M} a_k x_{n-k} + \sum_{j=1}^{N} b_j y_{n-j}$$

Output is linear combinantion of $M+1$ inputs x_k and N outputs y_j.

Special Cases

- Finite Impulse response (FIR) filter, all $b_j = 0$.
  $y_n = \sum_{k=0}^{M} a_k x_{n-k}$
  Non-recursive, output stops exactly M samples after the last input. Also called: Moving Average (MA) filter.
- Infinite Impulse Response (IIR) filter, some $b_j \neq 0$.
  Impulse response *may* be infinite.
  Auto Regressive (AR) filter if $M = 0$
- ARMA if $N \neq 0$ and $M \neq 0$

Digital Filtering in the Time-Domain

Most general linear digital filter formula:

$$y_n = \sum_{k=0}^{M} a_k x_{n-k} + \sum_{j=1}^{N} b_j y_{n-j}$$

Output is linear combinantion of $M + 1$ inputs x_k and N outputs y_j.

Special Cases

- Finite Impulse response (FIR) filter, all $b_j = 0$.
  $y_n = \sum_{k=0}^{M} a_k x_{n-k}$
  Non-recursive, output stops exactly M samples after the last input. Also called: Moving Average (MA) filter.
- Infinite Impulse Response (IIR) filter, some $b_j \neq 0$.
  Impulse response *may* be infinite.
  Auto Regressive (AR) filter if $M = 0$
- ARMA if $N \neq 0$ and $M \neq 0$

Digital Filtering in the Time-Domain

Most general linear digital filter formula:

$$y_n = \sum_{k=0}^{M} a_k x_{n-k} + \sum_{j=1}^{N} b_j y_{n-j}$$

Output is linear combinantion of $M + 1$ inputs x_k and N outputs y_j.

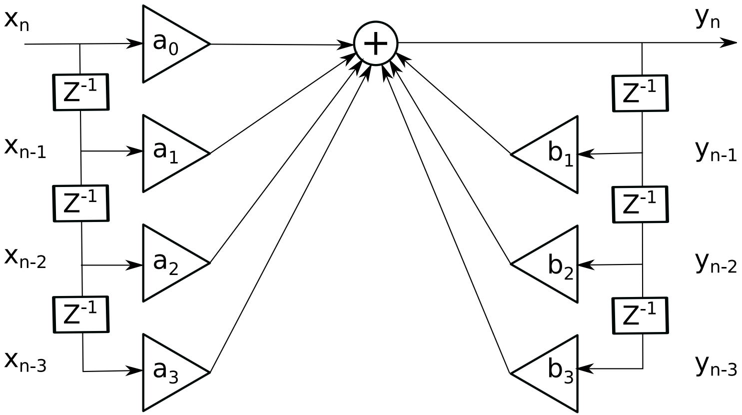
Special Cases

- Finite Impulse response (FIR) filter, all $b_j = 0$.
  $y_n = \sum_{k=0}^{M} a_k x_{n-k}$
  Non-recursive, output stops exactly M samples after the last input. Also called: Moving Average (MA) filter.
- Infinite Impulse Response (IIR) filter, some $b_j \neq 0$.
  Impulse response *may* be infinite.
  Auto Regressive (AR) filter if $M = 0$
- ARMA if $N \neq 0$ and $M \neq 0$

Digital Filter Layout Example

Layout of the filter:

$$
\begin{aligned}
y_n &= \sum_{k=0}^{3} a_k x_{n-k} + \sum_{j=1}^{3} b_j y_{n-j} \\
&= a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2} + a_3 x_{n-3} + b_1 y_{n-1} + b_2 y_{n-2} + b_3 y_{n-3}
\end{aligned}
$$



Digital Filter Frequency Response

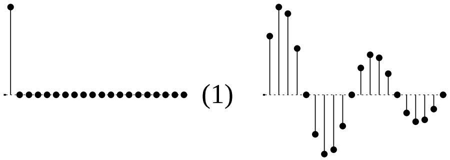
$$y_n = \sum_{k=0}^{M} a_k x_{n-k} + \sum_{j=1}^{N} b_j y_{n-j}$$

has :

$$Y(z) = \frac{\sum_{k=0}^{M} a_k z^{-k}}{1 - \sum_{j=1}^{N} b_j z^{-j}}$$

Numerator and denumerator are polynomials in z.
A polynomial may become zero for certain combination of its coefficients.
Numerator: no problem, i.e. $H(z_0) = 0$
Denumerator: unstable filter.

IIR Example: Formant Filter



$$
\begin{aligned}
y_n &= x_n - p y_{n-1} - q y_{n-2} \\
q &= e^{-2\pi BT} \\
p &= -2\sqrt{q}\cos 2\pi FT
\end{aligned}
$$

The Formant Filter $y_n = x_n - py_{n-1} - qy_{n-2}$

A second order recursive filter with response:

$$H(f) = \frac{1}{1 + pz^{-1} + qz^{-2}} = \frac{z^2}{z^2 + pz + q}$$

Denominator is second degree polynomial in z.

Zeros: $z_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

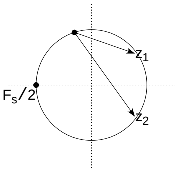
$$H(f) = \frac{1}{(z - z_1)(z - z_2)}$$

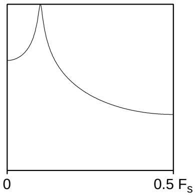
Interesting (resonance) when zeros lie within the "unit circle".

Then $\frac{p^2}{4} - q < 0$ and $z_{1,2} = -\frac{p}{2} \pm i\sqrt{q - \frac{p^2}{4}}$

We see that $z_1 = z_2^*$

The Formant Filter Frequency Response





Given p and q, z_1 and z_2 are fixed.
To get the frequency response $|H(f)| = \frac{1}{|z-z_1||(z-z_2|}$ we let z trace the upper part of the unit circle. When we start at $z = 0$ and end in $z = -1$ the response follows the curve on the right. When z is close to z_1, $|z - z_1|$ is very small and the response $\frac{1}{|z-z_1|}$ very large. We have resonance.
$z = 0$ means frequency $f = 0$.
$z = -1$ means $f = F_s/2$

Relations Between p, q and F, B

p and q and F

Solve $re^{i\phi} = z_1 = -\frac{p}{2} + i\sqrt{\frac{p^2}{4} - q}$

$\cos\phi = -\frac{p}{2\sqrt{q}}$

Because $\phi = 2\pi FT$ we get $F = \frac{1}{2\pi T} \arccos \frac{-p}{2\sqrt{q}}$

F, B and p, q

F & B (p,q):

$F = \frac{1}{2\pi T} \arccos \frac{-p}{2\sqrt{q}}$

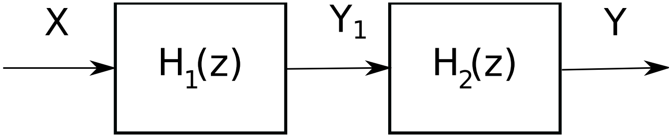
$B = \frac{1}{\pi T} \ln \sqrt{q}$

p & q (F, B):

$q = e^{-2\pi BT}$

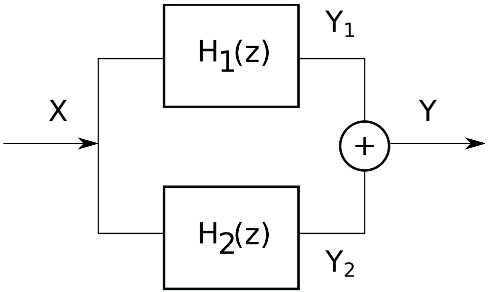
$p = -2\sqrt{q} \cos 2\pi FT$

More Formants: Serial



For the total filter response:
$H(z) = \frac{Y(z)}{X(z)} = \frac{H_2(z)Y_1(z)}{X(z)} = H_2(z)H_1(z)$
If $H_1(z)$ and $H_2(z)$ only have poles than $H(z)$ has only poles.

More Formants: Parallel



For the total filter response:

$H(z) = \frac{Y(z)}{X(z)} = \frac{Y_1(z)+Y_2(z)}{X(z} = H_1(z) + H_2(z)$

If $H_1(z)$ and $H_2(z)$ only have poles than $H(z)$ has poles and also may have zeros. These zeros depend on $H_1(z)$ and $H_2(z)$ and are fixed. This is not desireable.