

When one looks at a problem
from too many angles, one
runs the risk of seeing
nothing at all.

SOME REFLECTIONS ON REFLECTIONS
=====

by Hendrik Mol

In most textbooks on audiology the ossicular chain is referred to as a pressure transformer, being in fact the very successful result of a well-planned design objective of Nature: the high "impedance" of the "hard" inner ear fluid, the perilymphe, "has" to be "matched" to the low "impedance" of the air, the carrier of the acoustic messages. If not, one loosely argues, "harmful" reflections will occur at the ear drum rejecting acoustic energy meant to enter the cochlea.

The following notes tend to show that this traditional view needs some rethinking.

1. THE "CLASSIC" REFLECTIONS OF WAVES AT THE BOUNDARY OF TWO MEDIA

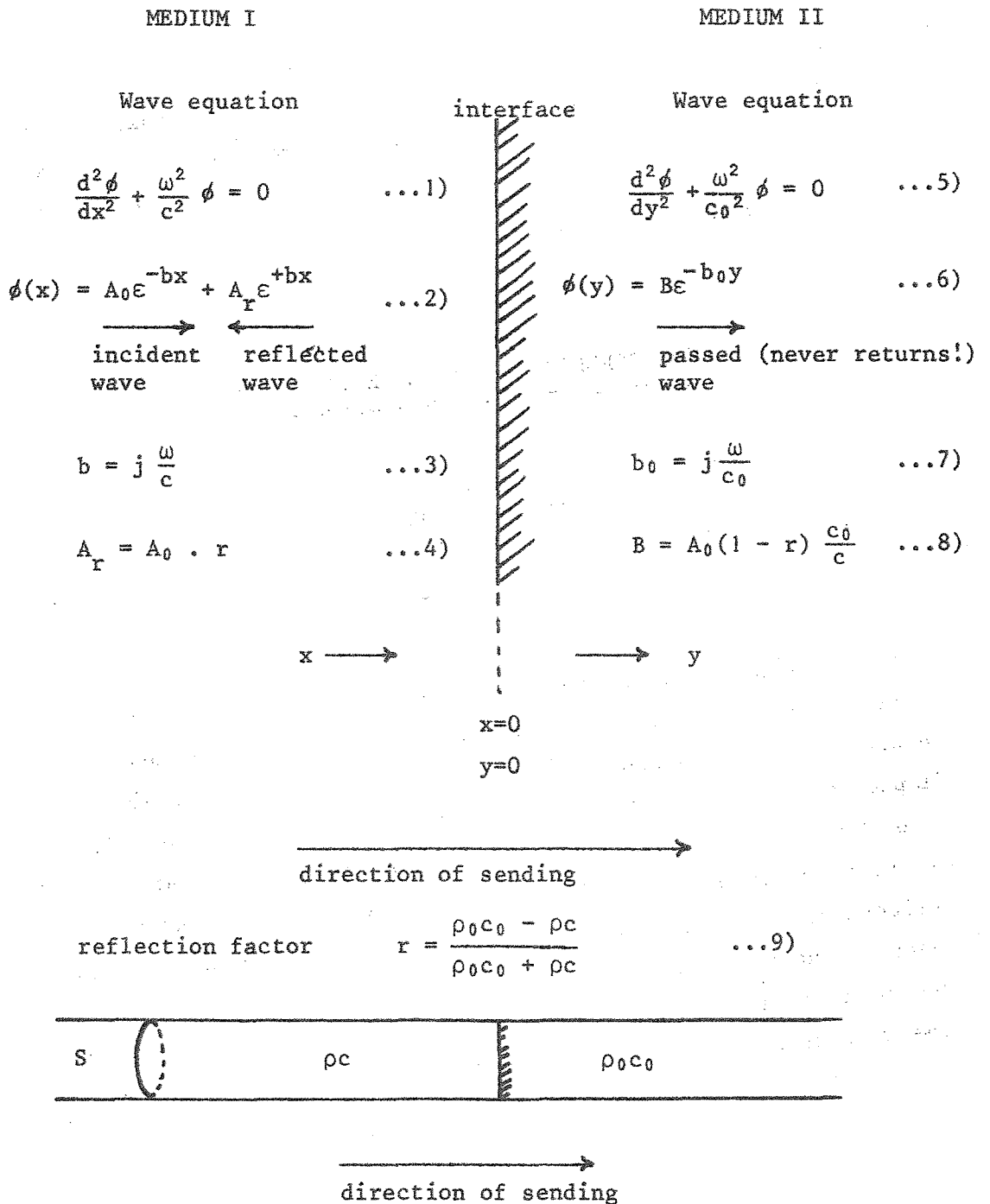


FIGURE I

The "classic" approach to the problem of the reflections at the boundary between two media.

Suppose, as is illustrated in Fig. I, that a plane in medium I, in our case air, perpendicularly enters the boundary plane with a second medium II, a watery fluid. We assume sinusoidal vibrations and use the velocity potential ϕ in our equations as a variable from which we may derive both the particle velocity u and the sound pressure p .

Medium I has the density ρ , the velocity of sound c whereas the position of a point in space is indicated by the coordinate x . Likewise, medium II has the density ρ_0 , the velocity of sound c_0 and the coordinate y . We introduce x and y in order to make a clearer distinction between vibrations in the two different media.

In medium I formula 1) represents the wave equation in terms of ϕ . In the solution 2) we can discern two waves:

The wave $A_0 \epsilon^{-bx}$ travels in the positive direction of x ; in other words, it travels towards medium II and may be considered as the incident "source" wave.

The wave $A_r \epsilon^{+bx}$ travels in the negative direction of x ; in other words, it comes from the boundary and may be considered as a reflection.

As seen from equation 4), a reflection factor r may be defined, fixing the relation between A_0 and A_r (without, however, saying anything about their absolute values!).

We shall now proceed to calculate r .

The particle velocity $u(x)$ may be derived from $\phi(x)$ in the following way:

$$u(x) = \frac{d\phi}{dx} \quad \dots 10).$$

The sound pressure $p(x)$ is given by:

$$p(x) = -j\omega\rho\phi \quad \dots 11).$$

In medium II we have:

$$u(y) = \frac{d\phi}{dy} \quad \dots 12)$$

and:

$$p(y) = -j\omega\rho_0\phi \quad \dots 13).$$

One always makes the tidy assumption that medium II is very vast and is reflectionless at the far end. This means, see Fig. I, that $\phi(y)$ only contains one travelling wave, as equation 6) clearly shows.

At the boundary the particle velocity must be continuous so that:

$$u(x=0) = u(y=0) \quad \dots 14).$$

By combining 2), 10), 6), 12) and 14) we finally arrive at:

$$b(A_0 - A_r) = b_0 B \quad \dots 15).$$

Also the sound pressure must be continuous which leads to:

$$p(x=0) = p(y=0) \quad \dots 16).$$

Combination of 2), 11), 6), 13) and 16) eventually yields:

$$\rho(A_0 + A_r) = \rho_0 B \quad \dots 17).$$

When we solve A_r and B from 15) and 17), also applying 3) and 7) we get:

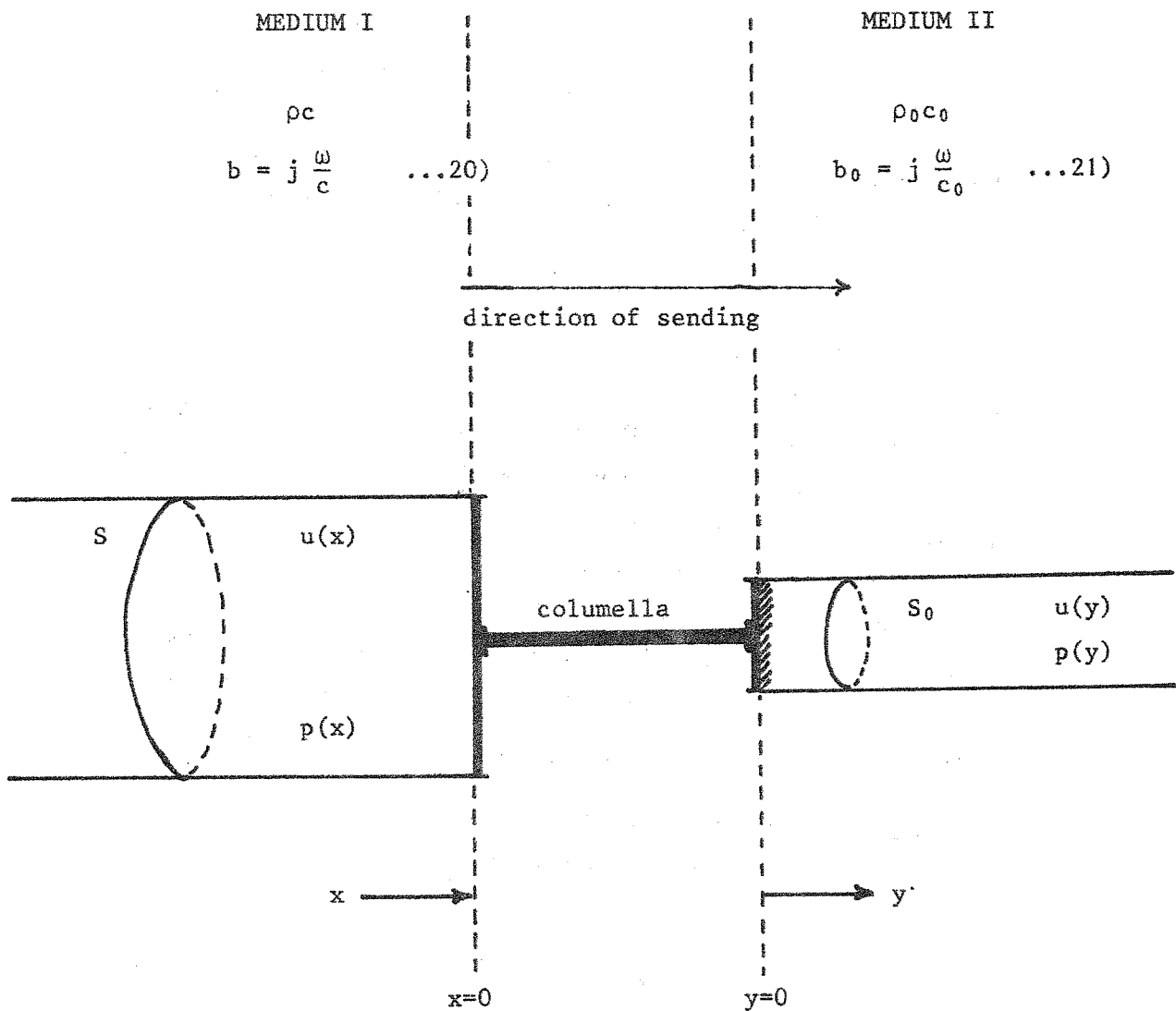
$$A_r = A_0 \frac{\rho_0 c_0 - \rho c}{\rho_0 c_0 + \rho c} \quad \dots 18)$$

and

$$B = A_0 \frac{2\rho c_0}{\rho_0 c_0 + \rho c} \quad \dots 19).$$

Equations 18) and 19) are in agreement with 4), 8) and 9) in Fig. I.

The product ρc (or $\rho_0 c_0$) has the dimension of a specific acoustic impedance (defined as sound pressure over particle velocity), coming to the fore in an elegant way in the reflection factor r , see also 9) in Fig. I. As, in general, the media will have different specific acoustic impedances, there will occur reflections at the boundary plane. If, for whatever reason, one wishes to avoid said reflections, introduction of a mechanical "matching" transformer capable of "adapting" ρc to $\rho_0 c_0$ comes to mind first. To this end it is necessary to have the media in tubes. As a first, faltering step we might put the media in one tube with constant cross area S . As all equations derived until now remain the same we do not seem to gain much by this tubing. Nevertheless, it brings conditions more in line with those met in the human ear, where the outer ear canal (external meatus) has the shape of a tube, whereas also the scala vestibuli is a (coiled up) tube filled with, among other things, perilymphe.



$$\phi(x) = A_0 \epsilon^{-bx} + A_R \epsilon^{+bx} \quad \dots 22)$$

$$\phi(y) = B \epsilon^{-b_0 y} \quad \dots 23)$$

$\xrightarrow{\text{incident wave}}$
 $\xleftarrow{\text{reflected wave}}$

$\xrightarrow{\text{passed wave}}$

$$u = \frac{d\phi}{dx} \quad \dots 24)$$

$$u = \frac{d\phi}{dy} \quad \dots 25)$$

$$p = -j\omega\rho\phi \quad \dots 26)$$

$$p = -j\omega\rho_0\phi \quad \dots 27)$$

$$A_R = A_0 \cdot r \quad \dots 28)$$

$$B = A_0 (1-r) \frac{c_0}{c} \quad \dots 29)$$

$$r = \frac{\rho_0 c_0 - \rho c \frac{S}{S_0}}{\rho_0 c_0 + \rho c \frac{S}{S_0}} \quad \dots 30)$$

FIGURE II

The columella transformer.

Fig. II shows how the situation changes radically when we give both tubes different cross areas and bound them by weightless pistons united by a stiff, likewise weightless rod we already now baptize "columella" in honour of the one and only ossicle of the bird.

The calculation of the reflection factor follows the same general pattern as before. At the columella boundary the particle velocity remains continuous:

$$u(x=0) = u(y=0) \quad \dots 31).$$

Because of the presence of the columella the forces on the pistons are equal: $S \cdot p(x=0) = S_0 \cdot p(y=0) \dots 32).$

By combining equations 20), 21), 22), 23), 24), 25), 26) and 27) with expressions 31) and 32) we are able to solve A_r and B which leads to the formulas 28) and 29) in Fig. II. The reflection factor r is given by 30).

It is interesting to notice how the ratio $\frac{S}{S_0}$ screws up the low product ρc to a higher value in order to bring it more in line with the high product $\rho_0 c_0$. In the most favourable case

$$\rho_0 c_0 = \rho c \frac{S}{S_0} \quad \dots 33)$$

so that $r=0$ and there are no reflections at the surface S, representing the tympanic membrane.

So far so good.

But how do the situations calculated so far relate to the human ear?

The situation depicted at the top of Fig. I reminds me of a man shouting at the fishes in a bottomless pond.

The tubular situation at the bottom of Fig. I seems to pertain to a man shouting down a deep well, the only echo originating at the surface of the water and not at the real sandy bottom of the well.

The columella transformer of Fig. II seems to come more in line with the human ear although it presupposes an infinitely long outer ear canal (external meatus) and can only boast of one ossicle (as we shall see later on these drawbacks can be overcome easily).

However, what is more serious, Fig. II supposes a very vast amount of fluid in the cochlea, an assumption being more or less ridiculed by the fact that the cochlea contains some 0.15 cm^3 (!) of perilymphe. On top of that comes that the simple wave equation 5) certainly does not hold for the incompressible viscous perilymphe and in fact does not take into account at all the mechanical behaviour of the cochlear partition, the ultimate receiver of the mechanical power pumped into the external ear! Interestingly enough, in the avian ear there is a damping pad behind the oval window, there being practically no perilymphe in the scala vestibuli. There is fluid in the scala tympani, however, (Schwartzkopff, 1968). Therefore we consider it worthwhile, if not imperative, to regard the stapes as a mechanical impedance Z_m and to perform a renewed calculation, albeit a simplified one, of the transmission properties of the ear, in an attempt to restate the problems of "matching", "transforming" etc., normally treated so smugly in literature. To avoid possible misunderstanding, we remind the reader of the definition of the mechanical impedance:

$$Z_m = \frac{\text{mechanical force on the stapes}}{\text{velocity of the stapes}} \quad \dots 34).$$

As A.S. Romer (Romer 1933, 1954) puts it:

"In fishes there was no special mechanism for transmitting sounds to the internal ear, which lay deeply buried within the braincase, but on land the problem of hearing is a very different one. Vibrations in the air are (except for something of the order of an explosion) too feeble to set up vibrations in the animal's body and reach the hearing organ in this fashion.

For the reception of air waves, tetrapods, from the early amphibians up, have established an amplifying mechanism (cf. p. 310). Across the tube of the old spiracle, and primitively in the notch mentioned above, is a membrane the eardrum, which picks up the sound waves. Between this membrane and an opening in the side of the braincase beneath it, which communicates with the internal part of the ear, there stretches in lower land types a small bone called the stapes, or stirrup; this is a modification, for a new use, of the hyomandibular bone which, in fishes, helps prop up the jaw joint."

In addition, in mammals the two jaw-joint bones have been pressed into service as accessory ossicles, the malleus and the incus. So, in the human ear there are three ossicles, in contrast to the avian ear which has only one ossicle, the columella.

It is interesting to notice (Gegenbauer) that also the jaw muscles have migrated towards the region of the ear; the m.stapedius can be traced back to the muscle attached to the original hyomandibular bone whereas the m.tensor tympani can be regarded as a portion of the original m.adductor mandibulae.

Says Homer: "...In mammals these old jaw bones have been taken over to complete the chain of three ear ossicles. We have, in these accessory hearing organs, one of the most interesting examples of change of function to be found in any animal. The cavities of the outer and middle ear were originally (like the spiracle) part of the fish-breathing apparatus. The malleus and incus were, in reptiles and lower forms, part of the jaw apparatus; the old jaw joint of our ancestors lies between two of the ossicles of our middle ear. The stapes was, in fishes, a prop for these old jaws. But both jaws and stapes were before this (among the jawless fishes) part of the primitive gill-bar structures. Breathing organs have become eating organs and then hearing organs. The function of these structures has changed radically; but their identity is unmistakable throughout.

Nature seldom makes new organs; but she is seldom wasteful and is adept at reshaping useless structures to fill new need."

More recently, Turmakin (1968) has stressed the fact that primitive terrestrial animals could never have been stone-deaf during the time it took to develop a mechanism for receiving air borne sound, because this would have led to extinction. He expresses the opinion that they helped themselves out with bone conduction, one of the possibilities being via the fore limb, as even to-day can be recognized in prostrate vertebrates.

So gradually the direct-route air sensitive ear took over auditory command until in Man it is the only practical way of receiving sound waves.

We are very much in sympathy with the broad lines of Romer's way of thinking. We see the middle ear as an improvisation of Nature. In general, improvisations seldom have the properties of careful planning or design: they just work and are not likely to be based on critical adjustments. We are even tempted to say that the indication "matching device" is a somewhat overdrawn beautification of a simple improvised driving pin for reaching deep-seated microphonic elements.

2. THE 'ANATOMY' OF A SIMPLE MICROPHONE

As a one-time designer of microphones for use in commercial telephone sets I was confronted with the rather trivial fact that the heart of a microphone is always a microphonic element of some sort that converts the mechanical movement of its driving pin, see Fig. III, into an electromotoric force. The electromotoric force e may be expressed, for instance, in the displacement x or the velocity v of the driving pin by means of a formula that at the same time states the sensitivity (in tele-

phony also called efficacy) of the element. As a matter of taste we prefer to use the velocity v though this choice does not necessarily imply that e must be proportional to v ; in millions of commercial microphones it is not.

In order to drive the pin it is inevitable to attach a diaphragm or membrane to it so that a force

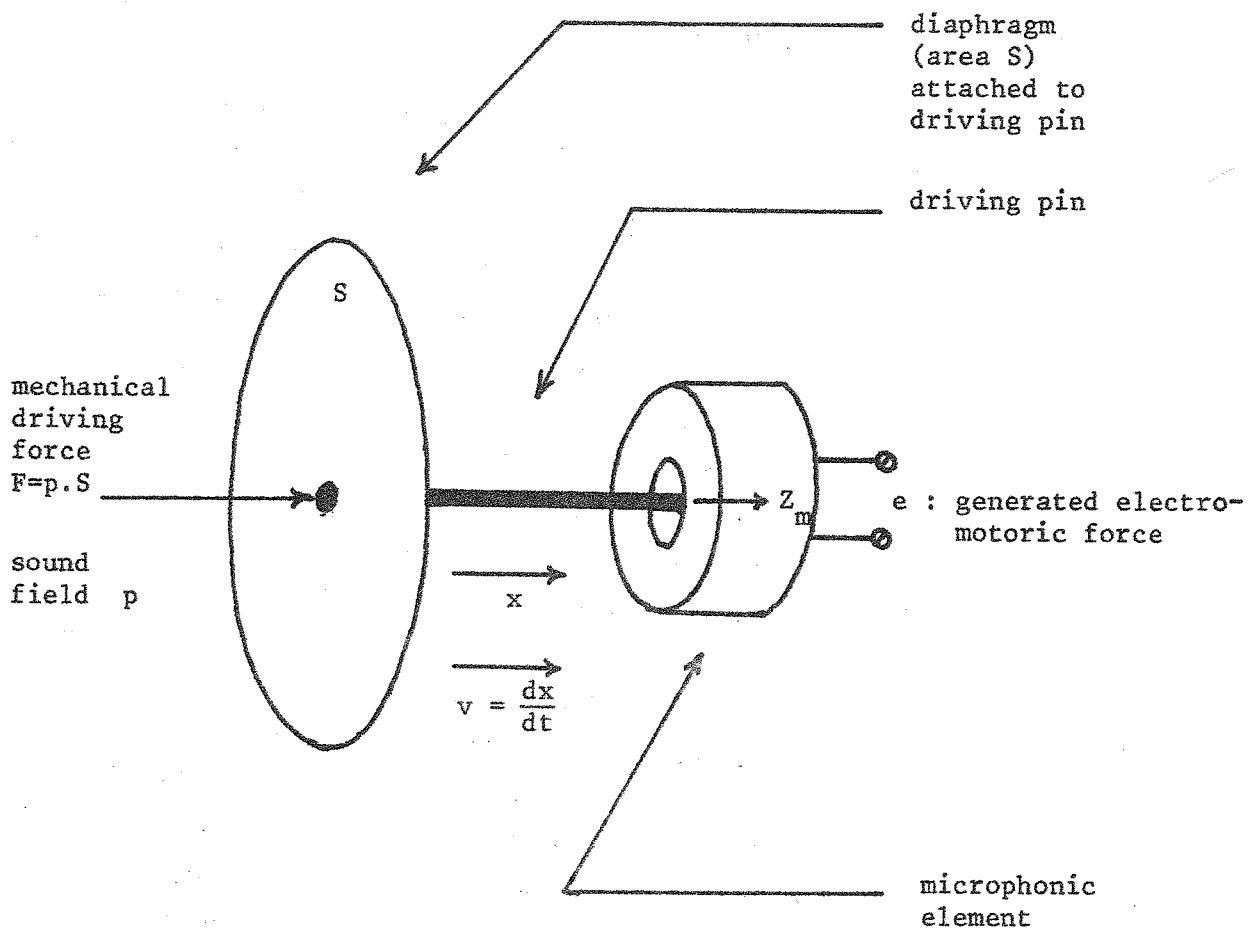
$$F = p \cdot S \quad \dots 35)$$

is collected from the sound field p , not unlike a sail in the wind.

In most cases I met in practice the mechanical input impedance Z_m (as defined in Fig. III) of the microphonic element was a given property, dictated by the very principle of the element, its stability, the small space allotted to it in the handset, and what have you. The only way to satisfy the, at that time, almost insatiable demand for "louder" (= more efficient) microphones was to increase the effective area S of the membrane, in order to raise v , in the hope of not decreasing the value of the sound pressure p presented by the talker.

It appears that the enlargement of S works out alright in the practical range available for changing S but is completely at variance with the modern trend towards small, light-weight microphone capsules in small, likewise light-weight handsets with flat mouthpieces, the cupped mouthpiece having fallen into disgrace for hygienic reasons. One is supposed to talk close to the flat mouthpiece in order to compensate for the too small diaphragm.

Our experience with commercial microphones raised our curiosity about the role of the value of the area of the ear drum in the human ear, as the quotient of that area and the area of the oval window, under the name of pressure transformation, is traditionally brought in relation with the so-called matching of the inner ear to the air.



x : displacement of driving pin

v : velocity of driving pin

$Z_m = \frac{p \cdot S}{v}$: mechanical impedance of microphonic element

$e = e(v)$

FIGURE III

Schematic representation of a microphone.

3. SIMPLIFIED CALCULATION OF THE VELOCITY v OF THE STAPES

In the traditional textbooks on audiology and phonetics reflections at the ear drum are ear-marked as unwanted phenomena that impede the function of hearing. It is held that Nature in its great wisdom has successfully combatted these reflections by "matching" the "impedance" of the cochlear fluid to that of the air by means of a "pressure transformation" achieved by the ossicular chain. Great value is laid on the ratio of the areas of the ear drum and the oval window.

In this paragraph we aim to work the other way round: calculate the all-important velocity v of the stapes (in a certain frequency region) and discuss how the modulus of this velocity depends on anatomical and physiological data like the area S of the ear drum, the length l of the ear canal, the mechanical lever ration β of the ossicular chain, the mechanical input impedance Z_m of the cochlea and frequency ω .

We shall look for respects in which a certain choice of the above mentioned five "variables" might be especially beneficial to the transport of power from the air to the inner ear. When such a favourable condition has been discovered (if present!) which, by the way, need not necessarily mean maximum power into the cochlea at a given frequency, we shall translate this condition in terms of reflections at the ear drum, expecting to find delicate degrees of difference from the traditional view.

In the traditional, crude matching model the mass and stiffness properties of the ear drum and the ossicular chain are neglected in the sense that they are seldom or ever mentioned; even in our in physical respect more realistic model we shall not take into account said properties either but we do furnish the motive: we limit ourselves to the frequency region of, say, 1000 - 5000 Hz, around the resonance frequency of the ossicular

chain. In other words, we assume that in this region the reactive part of the ossicular impedance plays a minor role in comparison with the high resistive (damping) component supplied by the cochlea. In case we venture ourselves outside this self-imposed restriction on the frequency range we shall explicitly state so.

The stapes is the mechanical entrance to the cochlea. Its movements are being hydraulically transferred to the microphonic elements in the cochlear partition.

The middle ear "sees" the cochlea as a mechanical impedance Z_m (as defined by 34)), into which it tries to "pump" a velocity v that, via an as yet unravelled mechanism, triggers off the nervous activities that lead to the various aspects of hearing.

Figure IV (1) is self-explaining.

The external meatus (outer ear canal) is represented by a tube with length l and constant cross-area S which is also considered as the area of the ear drum. The sound pressure at the entrance of the canal is supposed to be p , whereas the drum is supposed to be hit by the sound pressure p_t .

The set of equations 36) in Fig. IV 1 governs the ossicular chain.

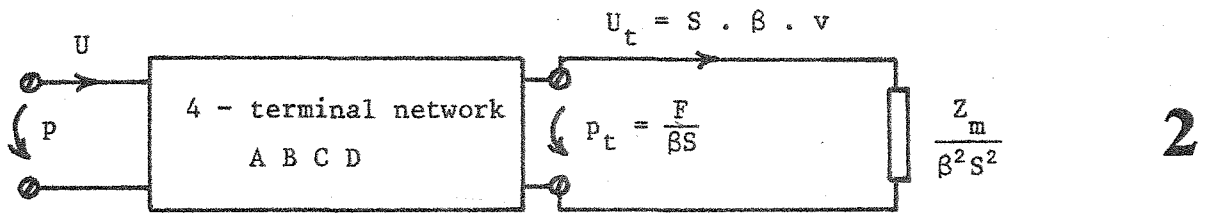
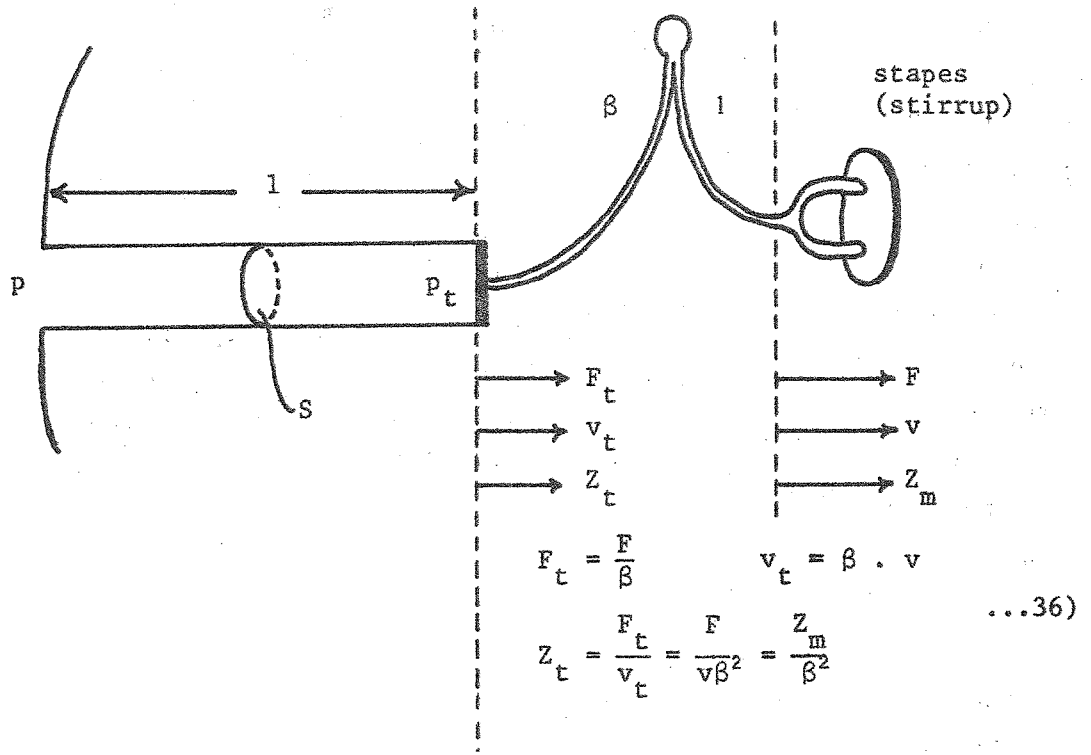
$F_t = p_t \cdot S$ is the force driving the ear drum,

v_t is the velocity of the ear drum, conveniently regarded as a piston,

F is the force driving the stapes,

v is the velocity of the stapes, whereas β is the lever ratio of the chain, often taken as about 1.3, a not over-dramatic value.

As shown in Figure IV 2, the external meatus is treated as an acoustical 4 - terminal network (see, for instance, Mol (1970, 1970)); p and the volume velocity U , sometimes also called flow or flux, the product of particle velocity and area, being the



4 - terminal representation of external meatus
 direction of sending

$$p = A p_t + B U_t \quad A = D = \cos \frac{\omega l}{c} \quad A.D. - B.C = 1$$

$$U = C p_t + D U_t \quad B = j \frac{\rho c}{S} \sin \frac{\omega l}{c} \quad C = j \frac{S}{\rho c} \sin \frac{\omega l}{c}$$

FIGURE IV

The calculation of the stapedial velocity v.

- 1 Schematical representation of the outer ear and the middle ear
- 2 Equivalent electrical circuit of 1. The 4 - terminal equations with their corresponding general circuit parameters have been indicated.

variables and A, B, C and D the general circuit parameters.

In order to nip in the bud possible misunderstandings, we shall enumerate below the three different types of "impedance" that are in current in the art:

$$\text{specific acoustic impedance} = \frac{\text{sound pressure}}{\text{particle velocity}} \quad \dots 37)$$

$$\text{acoustic impedance} = \frac{\text{sound pressure}}{\text{volume velocity}} \quad \dots 38)$$

$$\text{mechanical impedance} = \frac{\text{mechanical force}}{\text{velocity}} \quad \dots 39).$$

Many people, loosely talking about the ear, are quite expert at mixing up these three notions.

Starting from the simple truth that $F = v \cdot Z_m \quad \dots 40)$, we arrive, via 36), at the right-hand part of the equivalent circuit depicted in Figure IV 2, giving rise to the picture that at the end of the external meatus a sound pressure

$$p_t = \frac{F}{\beta S} \quad \dots 41)$$

$$\text{pumps a volume velocity } U_t = S \cdot \beta \cdot v \quad \dots 42)$$

$$\text{into an acoustic impedance } \frac{Z_m}{\beta^2 S^2} \quad \dots 43).$$

Obviously,

$$p_t = U_t \frac{Z_m}{\beta^2 S^2} \quad \dots 44).$$

Taking one of the 4-terminal equations given in Fig. IV 2 as a starting point:

$$p = A p_t + B U_t \quad \dots 45),$$

we may now proceed to the calculation of the stapedial velocity v . By combining 45), 42 and 44) we simply find:

$$v = \frac{p}{\beta S B + A \frac{Z_m}{\beta S}} \quad \dots 46).$$

After introducing the values for A and B as indicated in Fig. IV 2, we ultimately arrive at:

$$v = \frac{P}{j \beta \rho c \sin \frac{\omega l}{c} + \frac{Z_m}{\beta S} \cos \frac{\omega l}{c}} \quad \dots 47)$$

or, for short $v = \frac{P}{z} \quad \dots 48).$

The denominator of the fraction appearing in 47) has the dimension of a specific acoustic impedance. Because z links a sound pressure at the entrance of the external meatus to the velocity of the stapes at the far end of the ossicular chain, it may be called a "cross-impedance".

In order to study the influence of the denominator on the frequency-dependence of v , we need to know the modulus M of z . To simplify things, we assume with, for instance, Zwislocki (1948), that in the frequency range concerned Z_m is purely resistive and independent of frequency. Under these conditions we find that

$$M^2 = \rho^2 c^2 \beta^2 \sin^2 \frac{\omega l}{c} + \frac{Z_m^2}{\beta^2 S^2} \cos^2 \frac{\omega l}{c} \quad \dots 49).$$

Suppose that, by some "lucky hit" of Nature, the following relation holds:

$$\rho^2 c^2 \beta^2 = \frac{Z_m^2}{\beta^2 S^2} \quad \dots 50)$$

which can also be written as:

$$\frac{Z_m}{\beta^2 S^2} = \frac{\rho c}{S} \quad \dots 51) \quad \text{or} \quad Z_m = \rho c \beta^2 S \quad \dots 52).$$

When we combine 49) with 50) we simply arrive at:

$$M^2 = \rho^2 c^2 \beta^2 \left(\sin^2 \frac{\omega l}{c} + \cos^2 \frac{\omega l}{c} \right) = \rho^2 c^2 \beta^2 \quad \dots 53)$$

or: $M = \rho c \beta \quad \dots 54),$

independent of frequency!

In other words, the lucky hit, best expressed by 51), ensures a frequency independent transfer of power from p at the entrance of the ear canal to v , the input velocity of the cochlea. However, there is more to it: as is very clear from Fig. IV 2, 51) ensures that the ear canal is loaded by its characteristic acoustic impedance $\frac{\rho c}{S}$. Consequently, there will be no reflections at the ear drum! According to our model, this much praised condition is to be regarded as a secondary effect, accompanying a uniform frequency response.

The power W_c , fed by the stapes into the cochlea, is given by:

$$W_c = \frac{p^2}{M^2} \cdot Z_m = \frac{p^2}{\frac{\rho c}{S}} \quad \dots 55)$$

P being the effective value of p.

Are there any indications that such an assumption as the lucky hit might apply, albeit approximately, to the real human ear? Some friendly support comes from the almost classic equal loudness contours published by Fletcher and Munson (1933!), depicted below in Figure V.

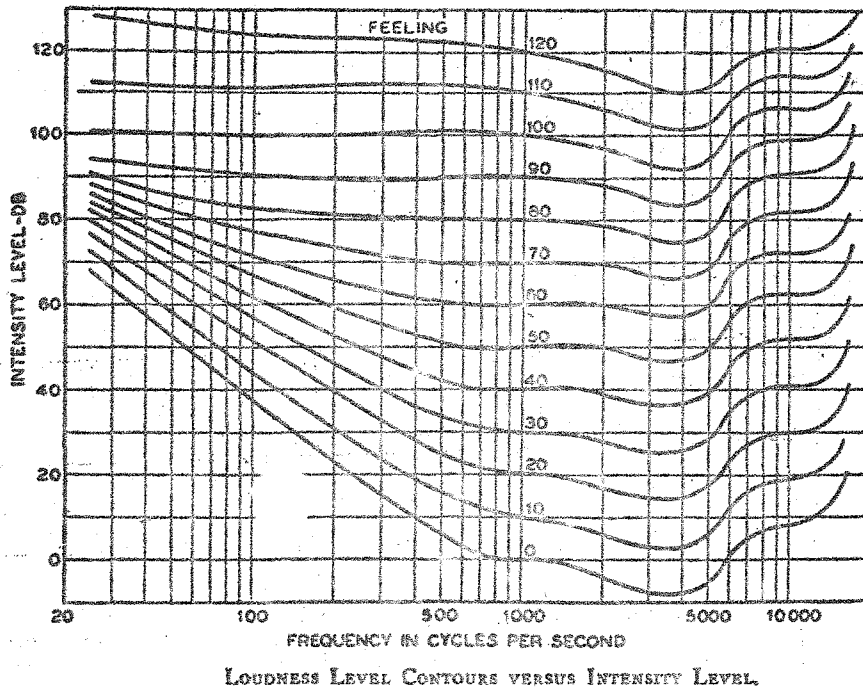


FIGURE V

The equal loudness contours.
(Fletcher and Munson, 1933)

Though the frequency characteristic of the stapedial velocity is not the only contribution by far to the shape of the equal loudness contours, these contours show an encouragingly horizon-

tal course in the frequency range from 800 - 5000 Hz, all curves being disfigured by a dip at about 3400 Hz. When we take $l = 2.5$ cm and $c = 34000$ cm/s we see, that the latter frequency corresponds to the quarter wavelength resonance of the external meatus. This feature is indeed displayed by formula 49) describing our model. When, in departure from 52), the lucky hit, we suppose Nature made a "near miss" by a factor m :

$$Z_m = m\rho c S \beta^2 \dots 56),$$

we may replace 49) by

$$M^2 = \rho^2 c^2 \beta^2 \left(\sin^2 \frac{\omega l}{c} + m^2 \cos^2 \frac{\omega l}{c} \right) \dots 57).$$

The following table is very convenient for interpreting ...57).

frequency f (Hz)	$\sin \frac{\omega l}{c}$	$\cos \frac{\omega l}{c}$	M^2
1700	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1+m^2}{2} \cdot \rho^2 c^2 \beta^2$
3400	1	0	$\rho^2 c^2 \beta^2$
5100	$\frac{1}{2} \sqrt{2}$	$-\frac{1}{2} \sqrt{2}$	$\frac{1+m^2}{2} \cdot \rho^2 c^2 \beta^2$

So the depth d (in dB) of the dip at 3400 Hz relative to the levels at 1700 Hz and 5100 Hz amounts to:

$$d = 10 \log \frac{1+m^2}{2} \dots 58).$$

Suppose we put the near miss at $m = 2$, corresponding to

$$Z_m = 2 \rho c \beta^2 \cdot S \dots 59)$$

then $d = 4$ dB $\dots 60).$

A glance at Figure V shows, that the educated guess for m being of the order of 2 does not seem to be too wild.

In the case of the near miss there indeed are reflections at the ear drum because the external meatus is loaded now by the acoustic impedance

$$\frac{\rho c}{S} \cdot m \quad \dots 61).$$

Said reflections result in a dip-like departure from a flat frequency characteristic of the stapedial velocity in the middle frequency range.

The power W delivered to the cochlea is given by

$$W = \frac{P^2}{M^2} Z_m = W_c \frac{m}{\sin^2 \frac{\omega l}{c} + m^2 \cos^2 \frac{\omega l}{c}} \quad \dots 62),$$

where W_c is the lucky hit power given by 55).

(For $m = 1$ the lucky hit situation re-appears)

If we again take m to be $m = 2$, then:

for $f = 3400$ Hz, $W = 2 W_c$, that is 3 dB better than the lucky hit,

for $f = 1700$ Hz or 5100 Hz, $W = \frac{4}{5} W_c$, which is 1 dB worse than the lucky hit. (The difference between these two situations is 4 dB, as it indeed should be.)

So, deviation from the flat frequency response means power gain (= better "match") for the resonance region but power loss for other frequencies. There are reflections at the ear drum at all frequencies concerned.

Let us now investigate the influence of variation of S on the stapedial velocity v .

A very drastic way of starting to satisfy our curiosity is to make $l = 0$, which comes down to having, frog-fashion, the ear drum at the surface of the head. In that case formula 47) on page 16 yields:

$$v = \frac{\beta S}{Z_m} p \quad \dots 63).$$

This formula we find back in Figure III, in a slightly different

form and with $\beta = 1$, where it applies to a simple commercial microphone.

We shall now return to the human ear with its 2.5 cm long external meatus and study how variations in S (achieved in a pipe-dream or, less likely, by scheming Nature) might affect M , the modulus of z . We must realize, however, that even in a brain phantasy we are not allowed to over-increase S because in that case the one-dimensional treatment of the waves in the external meatus is at stake. Obviously, for very high values of S , the ear drum is at the surface of the head again, frog-fashion, so that 63) comes into action! As a matter of fact equation 49) is valid in a limited range of the value of S only. By the way, Nature's freedom for experiments (if any!) with the value of S should not be over-estimated in view of the fact that the meatus is to be regarded as an abandoned gill slit in a skull of limited dimensions.

From 49) it is clear that M always decreases when S is being enlarged; consequently the stapedial velocity will always increase. Enlargement of S only seems to have practical limits.

4. THE INFLUENCE OF VARIATION Z_m ON THE POWER W DELIVERED TO THE COCHLEA

Up to now in our lucky hit (and near miss) approach Z_m was regarded as being real, positive and independent of frequency. We might try to make the case more general by allowing Z_m to be real, positive and, for a change, dependent on frequency.

We might then put the academic question (though, at heart I dislike using the word academic for such a silly question) : given S , l , β and ω , how might we (or Nature) choose Z_m so that maximal power W is developed in Z_m ? This, after all, is the famous question asked by the "matchmakers".

A simple way of answering this question is to start from the near miss notation for Z_m :

$$Z_m = mpc\beta^2 \quad \dots 56)$$

and to saddle m with the task of variable. Then we merely have to turn to 62) and to determine the (partial) derivation of W with respect to m , declaring it equal to zero:

$$\frac{\partial W}{\partial m} = 0 \quad \dots 64).$$

Formula 64) leads to the condition, that:

$$m = \pm \operatorname{tg} \frac{\omega l}{c} \quad \dots 65).$$

So indeed, m appears to be a function of frequency. Driven by economy, for every value of ω we may find a value of Z_m (to be arrived at by Nature) into which, at that particular frequency, maximal power is being fed. We find this power, W_m , by introducing 65) in 62), which leads to:

$$W_m = \pm \frac{W_c}{2 \cdot \sin \frac{\omega l}{c} \cos \frac{\omega l}{c}} = \pm \frac{W_c}{\sin 2 \frac{\omega l}{c}} \quad \dots 66).$$

Let us call the value of m given by 65), the best (power) match at the frequency ω . A glance at the following table, based on 65),

f (Hz)	m
1133	0,58
1700	1,00
2266	1,73
3400	∞
4532	1,73
5100	1,00

shows us, that, in the range from 2266 - 4572 Hz, the frequency behaviour of m , and thus of $Z_m = mpc\beta^2$ is very stormy. (We should not quibble about the preposterous value of $m = \infty$ at

3400 Hz because this is no doubt the result of one of the simplicities of our model that, among other things, does not allow energy losses in the external meatus. At any rate, m may be supposed to be very high at 3400 Hz, the quarter wavelength resonance of the external meatus.) As present day estimations of Z_m (we already mentioned Zwislocki, 1948) assume Z_m to be independent of frequency, it is highly improbable that the "best match" principle must be considered as a possible design objective for "constructing" the ossicular chain.

It is interesting to notice, by the way, that the best matches enumerated in the table, are all accompanied by reflections at the ear drum, except at the frequencies that are odd multiples of 1700 Hz. For these frequencies, the corresponding $m = 1$ ensures a load of $\frac{\rho c}{S}$ at the drum end of the external meatus, in that way guaranteeing the absence of reflections!

5. THE REFLECTION FACTOR r AT THE EARDRUM

In the external meatus the equations 20), 22), 24), 26) and 28) are still valid, describing the picture of a wave travelling in the direction of the drum, having an amplitude proportional to A_0 and a travelling wave coming from the drum with an amplitude proportional to A_r .

At the eardrum we have, see Fig. IV 2:

$$\frac{p(x=0)}{S \cdot u(x=0)} = \frac{Z_m}{\beta^2 S^2} \quad \dots 67).$$

Combination of 20), 22), 24), 26), 28) and 67) finally yields:

$$Z_m = \rho c S \beta^2 \frac{1 + r}{1 - r} \quad \dots 68),$$

or:

$$r = \frac{Z_m - \rho c S \beta^2}{Z_m + \rho c S \beta^2} \quad \dots 69).$$

Introduction of the near miss factor m as defined in 56) leads to simply:

$$r = \frac{m - 1}{m + 1} \quad \dots 70).$$

When m lies between 2 and 3, then r will lie between $\frac{1}{3}$ and $\frac{1}{2}$.

6. THE ROLE OF THE PRESSURE TRANSFORMATION IN THE REFLECTION FACTOR r

Traditionally, the pressure transformation γ is quite often declared to be proportional to the quotient of the area S of the drum and the area S_0 of the oval window:

$$\gamma = \frac{S}{S_0} \cdot \beta \quad \dots 71).$$

One tacitly assumes, that the force exerted on the drum by the vibrations in the external meatus is being transferred to the stapes uninfluenced by the mass, stiffness and resistance properties of the ossicular chain. In the present paper we do not take into account said properties either but we motivate this by restricting our frequency range. Because we work with Z_m , the area S_0 does not appear in 69). We have to drag it in by the head and shoulders by putting:

$$Z_m = z_c \cdot S_0 \quad \dots 72),$$

where z_c is defined as the specific acoustic input impedance of the cochlea.

By combining 69), 71) and 72) we ultimately arrive at:

$$r = \frac{\frac{z_c}{\beta} - \rho c \gamma}{\frac{z_c}{\beta} + \rho c \gamma} \quad \dots 73).$$

Zwislocki (1948) has estimated z_c and arrived at the following formula (we changed the notation for practical reasons):

$$z_c = \sqrt{\frac{2\rho_c S_0}{C_0}} \quad \dots 74),$$

where: ρ_c is the density of the perilympe
and C_0 the volume elasticity of the cochlear parti-
tion.

For those who prefer to cling to the notion that the pressure transformation \mathcal{Z} as such is a clever method "applied" by Nature in order to "adapt" the "low" ρ_c of the air to the "high" z_c (divided by β) of the cochlear contents in order to reduce reflections at the ear drum, there are some interesting details to chew on. A certain value of \mathcal{Z} may be achieved in many ways because \mathcal{Z} is a function of

S , S_0 and β . As $\frac{z_c}{\beta}$ is a function of S_0 and β , it seems to be more realistic to determine the separate influences of S , S_0 and β or r instead of regarding \mathcal{Z} as a variable.

SUMMARY AND CONCLUSIONS

The simplified model of the sound conducting system of the outer and middle ear is meant to be valid for the frequency range of, say, 1000 - 5000 Hz.

A simple relation can be found between the stapedial velocity v and the sound pressure p at the entrance of the external auditory meatus. The specific acoustical cross-impedance of the system may be defined as

$$z = \frac{p}{v} \quad \text{from: 48).}$$

In the cross-impedance z the mechanical input impedance Z_m of the cochlea plays an important role. If we assume, in the wake of, for instance, Zwislocki (1948) that, in the frequency range concerned, Z_m is purely resistive and independent of frequency, there appears to be an elegant value of Z_m , to wit:

$$Z_m = \rho c S \beta^2 \dots 52),$$

playfully indicated by us as the "lucky hit" of Nature. When Z_m has the lucky hit value given by 52), the modulus M of z becomes independent of frequency. In other words, the amplitude relation between p and v becomes frequency independent! Calculation shows that in the case of the lucky hit the external meatus is terminated at its drum end by its characteristic acoustic impedance:

$$\frac{\rho c}{S} \dots 51).$$

Consequently, there will be no reflections at the ear drum. This Utopian condition, however, accompanies a flat frequency response. It does not guarantee that, in the frequency range concerned, the impedance Z_m of the cochlea derives optimal power from the external meatus.

It would, of course be too beautiful a coincidence that 52) would be exactly obeyed by Z_m . We suppose that a "near miss" has been made by a factor m , defined as follows:

$$Z_m = m \rho c S \beta^2 \dots 56)$$

For this near miss situation our model predicts a deviation from the flat frequency response with a peak at 3400 Hz, the quarter wavelength resonance of the external meatus. The height of this peak, in dB relative to the values at 1700 and 5100 Hz, is given by:

$$d = 10 \log \frac{1+m^2}{2} \dots 58).$$

So when, as an educated guess, we suppose m lies between 2 and 3, the corresponding d lies between 4 and 7 dB, in my opinion a not too far fetched correspondence with the ossicular frequency response, one of the factors underlying the shape of the equal loudness contours (Fletcher - Munson, 1933).

As soon as $m > 1$, the frequency response will no longer be flat,

there will be reflections at the ear drum, but, nevertheless, in the region around 3400 Hz, the power derived by the cochlea from the external meatus may be higher than in the lucky hit situation.

If, as first predicted by Zwislocki (1948), the mechanical impedance Z_m of the cochlea is purely resistive and frequency independent, as we also assume in our simple model, then there would be a very interesting consequence, because the following "design objective" of the ear would no longer hold water: the ear is so designed that in the relevant frequency range optimal power is derived from the external meatus by the cochlea. Calculation shows that for meeting this requirement a Z_m is needed that varies considerably with frequency, which is not the case.

In other words: "power matching" in the human ear is less important than improvement of the frequency characteristic.

In our simple model it remains possible to define a reflection factor r at the ear drum, but in this factor the pressure transformation does no longer play the solo role hinted at by tradition.

REFERENCES

1. Romer, A.S.; Man and the Vertebrates, (1933), (1954),
Pelicans 303 and 304.
2. Zwislocki, J. - Moscicki; Theorie der Schneckenmechanik, (1948),
Thesis Zürich.
3. Mol, H.; Fundamentals of Phonetics, II, (1970),
The Hague-Paris, Mouton.
4. Mol, H.; Paradoxes around the exponential horn, Proceedings
of the Institute of Phonetic Sciences, (1970),
Amsterdam.
5. Gegenbauer, C.; Vergleichende Anatomie der Wirbeltiere.
6. Turmarkin, A.; Evolution of the auditory conducting apparatus
in terrestrial vertebrates, (1968), Hearing Mecha-
nisms in Vertebrates, A Ciba Foundation Symposium,
London, J. & A. Churchill Ltd.
7. Schwartzkopff, J.; Structure and function of the ear and of
the auditory brain areas in birds, (1968), Hearing
Mechanisms in Vertebrates, etc.
8. Harvey Fletcher and W.A. Munson, Loudness, Its Definition,
Measurement and Calculation, J.A.S.A., (1933),
Volume V, pp 82 - 108.
9. Proposed Standards for Noise Measurement (As Adopted June 26,
1933, by The American Standards Association Committee on
Acoustical Measurements and Terminology), J.A.S.A. (1933),
Volume V, pp 109 - 111.

APPENDIX

The myth of the reflected power at the ear drum

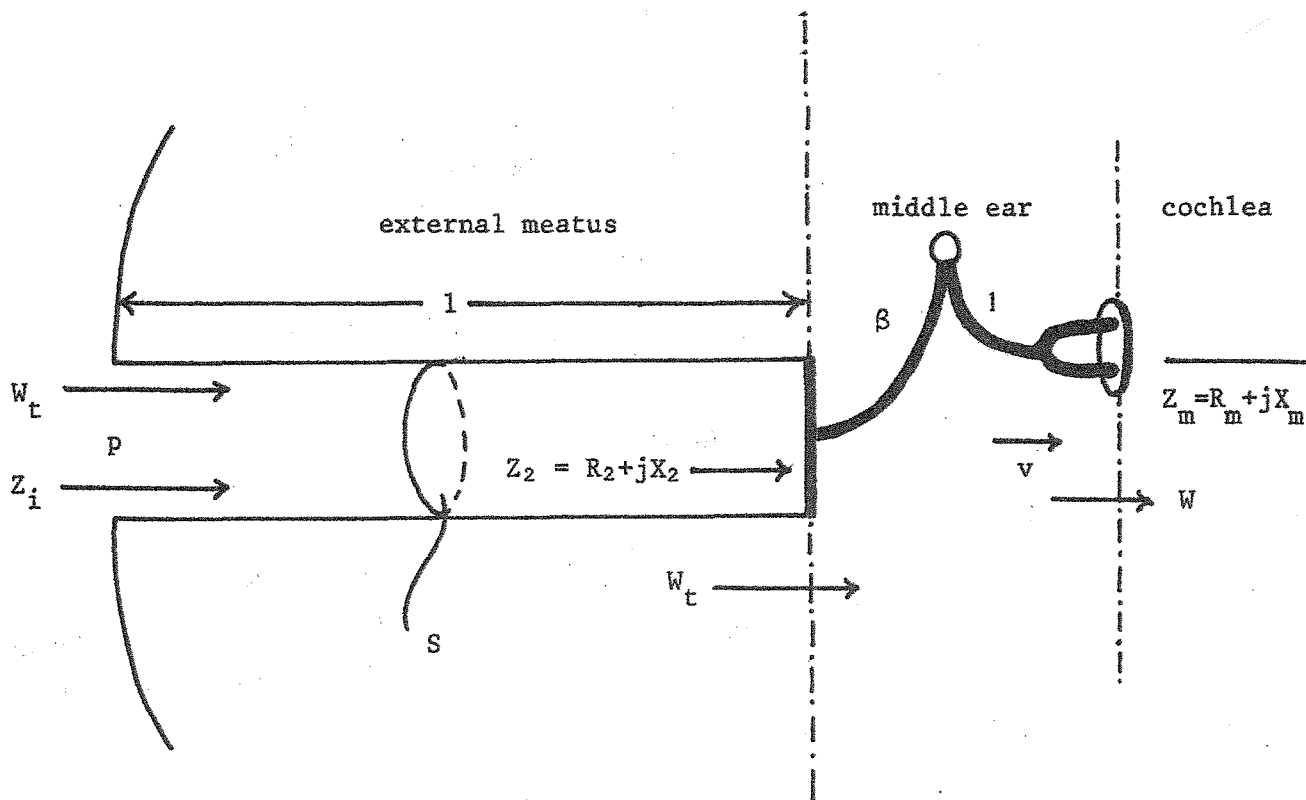


FIGURE VI

The power transfer from the outer space to the cochlea. Obviously:

$$W_t = W$$

In our model the external meatus is thought to be loss free. This means it cannot consume power. (Also in the human ear the power drain in the meatus may be neglected.) Consequently, the acoustic power W_t entering the meatus at the tragus is fed into the middle ear in toto. When, as we did up to now in our calculations, we also neglect the power losses in the middle ear, W_t

is being completely consumed by the cochlea. When we define W as the power dissipated in the cochlear impedance Z_m , then, obviously $W_t = W$...75).

It is possible to verify 75) by calculation, though this merely comes down to proving that $l = 1$. We shall nevertheless perform this calculation because it might help the reader to better understand the mechanism of power transfer in the ear.

Let us first make clear what is meant exactly by the power W_t pumped by the sound pressure p into the external meatus. It has the dimension of an energy per second, though many radio people and even physicists loosely speak about energy, ignoring the "per second". W_t is the average of the instantaneous value of the power, labelled w_t , taken over the period T of sinusoidal p :

$$W_t = \frac{1}{T} \int_0^T w_t dt \quad \dots 76).$$

The so-called effective value (root mean square value) of any sinusoidal function of time $f(t)$ is indicated by the symbol F and is defined as follows:

$$F^2 = \frac{1}{T} \int_0^T [f(t)]^2 dt \quad \dots 77).$$

It is a well-known fact, see Fig. VI, that

$$W_t = (\text{effective value of } p) \times (\text{effective value of } U) \times \cos \psi_i \quad \dots 78)$$

where ψ_i is the argument of complex Z_i , and U is the volume velocity at the entrance of the meatus.

This can be written as follows:

$$W_t = P^2 \times \text{real part of } \frac{1}{Z_i} \quad \dots 79).$$

Figure IV contains the formulas for determining $\frac{1}{Z_i}$.

We simply have to introduce $p_t = U_t \cdot Z_2$ and to elaborate:

$$\frac{1}{Z_i} = \frac{U}{P} = \frac{C \cdot Z_2 + D}{A \cdot Z_2 + B} \quad \dots 80).$$

After elaboration we arrive at:

$$\text{real part of } \frac{1}{Z_i} = \frac{R_2}{R_2^2 \cos^2 \frac{\omega l}{c} + (X_2 \cos \frac{\omega l}{c} + \frac{\rho c}{S} \sin \frac{\omega l}{c})^2} \quad \dots 81),$$

it, of course, being understood that $Z_2 = R_2 + jX_2$...82).

The power W , see Fig. VI, pumped into the cochlea, is given by:

$$W = V^2 \cdot R_m \quad \dots 83),$$

where V is the effective value of the stapedial velocity v and R_m is the real part of the mechanical input impedance Z_m of the cochlea, given by

$$Z_m = R_m + jX_m \quad \dots 84).$$

For this calculation it is not even necessary to consider, as we did up to now, Z_m as purely resistive.

From 47) and 48) we see, very easily, that

$$V^2 = P^2 \frac{1}{|z|^2} \quad \dots 85)$$

with
$$z = \frac{R_m}{\beta S} \cos \frac{\omega l}{c} + j \left(\frac{X_m}{\beta S} \cos \frac{\omega l}{c} + \beta \rho c \sin \frac{\omega l}{c} \right) \quad \dots 86).$$

We already know, see Fig. IV, that the mechanical impedance Z_m may be transformed into an acoustical impedance Z_e loading the end of the external meatus by dividing it by $\beta^2 S^2$, so that:

$$Z_2 = \frac{Z_m}{\beta^2 S^2} = \frac{R_m}{\beta^2 S^2} + j \frac{X_m}{\beta^2 S^2} = R_2 + j X_2 \quad \dots 87).$$

Combination of 85), 86) and 87) finally yields, after some elaboration:

$$W = P^2 \frac{R_2}{R_2^2 \cos^2 \frac{\omega l}{c} + (X_2 \cos \frac{\omega l}{c} + \frac{\rho c}{S} \sin \frac{\omega l}{c})^2} \quad \dots 88).$$

This is, quite expectedly, exactly the same value we found for W_t , see 79) and 81).

As I see it, the 4-terminal method, as illustrated in Fig. IV (Mol 1970, 1970) is the most simple way to determine the relation between the stapedial velocity v and the sound pressure p at the entrance of the external meatus. After all, it is the stapedial velocity we want to know. No doubt, further studies of the cochlea must reveal the relation between v and the nervous activities in the eighth nerve, whereas, of course, also the mechanical input impedance Z_m of the cochlea must be determined with the aid of convincingly adequate methods.

The 4-terminal method permits us to calculate in a simple way the power W_t pumped into the ear by p ; no premature mention is made of "reflections" at the ear drum. The 4-terminal method is completely "legitimate" because its equations have also been derived from the velocity potential ϕ containing the constants A_0 and A_r which, appearing in the two terms of ϕ , masquerade as amplitudes of two waves travelling in opposite directions. This picture is obscured by the 4-terminal method which, in an elegant way, provides the relation between the twin quantities p and U at the entrance of the meatus and the pair of p and U at the drum end.

Formula 75) shows the trivial fact that all power entering the meatus is being transported to the cochlea, without any deduction, so that whatever quantity may be "reflected" at the ear drum, it is certainly not the power.

When, for whatever reason, the cochlea does not receive enough power, we should not blame power reflections at the drum for that. The real trouble lies in the fact that already at the entrance of the meatus too low power is admitted due to an uninviting value of the acoustic input impedance the meatus presents to the sound pressure p prevailing there.

The notion "reflection" is defined in terms of the relation between the travelling waves forming the two terms of the velocity

potential ϕ . When $A_r = 0$, there will be no "reflected" term in ϕ (see 22) in Fig. II). Then merely the "incident" wave travelling into the directions of the drum remains. As our calculations showed, this tidy situation does not guarantee optimal power dissipated in the cochlea: it ensures a frequency independent modulus of the cross-impedance linking p to v . There are more ways than one to decompose the velocity potential into terms.

As said before, 22) is valid for the external meatus:

$$\phi = A_0 \epsilon^{-bx} + A_r \epsilon^{bx} \quad \dots 22).$$

It is always possible to define a reflection factor r as follows

$$A_r = r \cdot A_0$$

with

$$r = \frac{Z_m - cS\beta^2}{Z_m + cS\beta^2} \quad \dots 69).$$

After consulting 87) we may write ...69) as follows:

$$r = \frac{Z_2 - \frac{\rho c}{S}}{Z_2 + \frac{\rho c}{S}} = |r| \epsilon^{j\theta_r} \quad \dots 89)*$$

at the same time attacking the general case that Z_m is complex.

Also A_0 will be complex:

$$A = |A_0| \epsilon^{j\theta_0} \quad \dots 90).$$

We know already that:

$$b = j \frac{\omega}{c} \quad \dots 20).$$

It is simple to show that it is allowed to write ...22) as follows:

$$\phi = A_0 [\epsilon^{-bx} - r \epsilon^{-bx} + r (\epsilon^{bx} + \epsilon^{-bx})] \quad \dots 91).$$

* The formulas 89) up to and including 100) remain valid even if we do no longer ignore the masses, stiffness and resistances in the ossicular chain.

By taking the trouble of advancing one small step by introducing 20), we arrive at:

$$\phi = A_0 \left[\epsilon^{-j\frac{\omega x}{c}} - r\epsilon^{-j\frac{\omega x}{c}} + 2r \cos \frac{\omega x}{c} \right] \dots 92).$$

The complete expression for the velocity potential is a function of x and t:

$$\psi(x,t) = \phi(x)\epsilon^{j\omega t} \dots 93).$$

By combining 93), 92), 90) and 89) we get the following result:

$$\psi(x,t) = |A_0| \left[\epsilon^{j[\omega(t-\frac{x}{c})+\theta_0]} - |r|\epsilon^{j[\omega(t-\frac{x}{c})+\theta_0+\theta_r]} + 2|r|\cos\frac{\omega x}{c}\epsilon^{j(\omega t+\theta_0+\theta_r)} \right] \dots 94).$$

This expression may be simplified as follows:

$$\psi(x,t) = |A_0| \left[C\epsilon^{j[\omega(t-\frac{x}{c}) + \theta_0 + \delta]} + 2|r|\cos\frac{\omega x}{c}\omega^{j(\omega t + \theta_0 + \theta_r)} \right] \dots 95),$$

where δ and C are determined by:

$$C \sin \delta = -|r| \sin \theta_r \dots 96)$$

$$C \cos \delta = 1 - |r| \cos \theta_r \dots 97)$$

$$C = \sqrt{1 + |r|^2 - 2|r|\cos\theta_r} \dots 98),$$

28), 29) and 30) forming the consequence of:

$$1 - |r| \epsilon^{j\theta_r} = C \epsilon^{j\delta} \dots 99).$$

Coming down to earth, we take the real part of $\psi(x,t)$, which leads to:

$$|A_0| \left[\underbrace{C \cos [\omega(t-\frac{x}{c}) + \theta_0 + \delta]}_{\text{travelling wave} \rightarrow x} + 2|r|\underbrace{\cos \frac{\omega x}{c} \cos (\omega t + \theta_0 + \theta_r)}_{\text{stationary wave}} \right] \dots 100)$$

describing the velocity potential that governs the goings on in the external meatus.

Equation 100) presents the following picture (which does not "bite" the 4-terminal method at all!) of the general case of vibrations in the external meatus: there is a travelling wave running towards the drum, superimposed on a standing wave pattern. One may take this picture or leave it.

In any case 100) may be used for generating the velocity potentials pertaining to several interesting ways of loading the meatus at the drum end.

In Figure VII some examples are given.


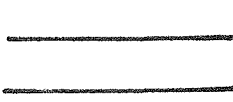
	real part of $\psi(x,t)$	Z_2	r	θ_r	γ	C	
1	$ A_0 \cos[\omega(t - \frac{x}{c}) + \theta_0]$ $\xrightarrow{\quad\quad\quad} x$ travelling wave	$\frac{\rho c}{S}$	0	0	0	1	no reflections at the drum
2	$2 A_0 \cos \frac{\omega x}{c} \cos(\omega t + \theta_0)$ stationary wave	$-j\omega$	1	0	0	0	
3	$2 A_0 \sin \frac{\omega x}{c} \sin(\omega t + \theta_0)$ stationary wave	0	1	π	0	2	

FIGURE VII

Some examples of possible velocity potentials

- Case 1: The meatus is loaded with its characteristic acoustic impedance
- Case 2: The drum is completely stiff
- Case 3: the drum presents a zero impedance (no matter how this might be accomplished physically).